LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FOURTH SEMESTER – APRIL 2010

MT 4502/MT 4500 - MODERN ALGEBRA

Date & Time: 21/04/2010 / 9:00 - 12:00 Dept. No. Max. : 100 Marks

PART – A

Answer **ALL** the questions (10 x 2 = 20 marks)

1. Let P be the set of positive integers and let $a \le b$ mean that a divides b. Prove that P is a partially ordered set.
2. If G is a group then prove that the identity element of G is unique.
3. Prove that every cyclic group is abelian.
4. Prove that every subgroup of an abelian group is normal.
5. Expres (1,3,5) (5,4,3,2) (5,6,7,8) as a product of disjoint cycles.
6. Show that the additive group G of integers is isomorphic to the multiplicative group G' = $\{\ldots 3^{-2}, 3^{-1}, 3^0, 3^1, 3^2, \ldots\}$.
7. If F is a field then prove that its only ideals arc (O) and F itself.
8. If A is an ideal of a ring R with unity and $1 \in A$ then prove that A = R.
9. Let R be a Euclidean ring and suppose that for a,b,c, $\in$ R, $a \mid bc$ and that a and b are relatively prime. Then prove that $a \mid c$.
10. Define a maximal ideal of a ring.

PART – B

Answer any **FIVE** questions (5 x 8 = 40 marks)

11. Prove that $(ab)^2 = a^2b^2$ for all a,b in a group G if and only if G is abelian.
12. If H and K are any two non empty subsets of a group G then prove that $(HK)^{-1} = K^{-1} H^{-1}$.
13. Prove that subgroup N of a group G is a normal subgroup of G if and only if the product of two left cosets of N in G is again a left coset of N in G.
14. Suppose a and b are elements of a group and $a^2 = e$, $b^6 = e$, $ab = b^4a$. Find the order of ab.
15. Prove that every group is isomorphic to a group of permutations.
16. Let f be a homomorphism of a group G into a group G′ (i) If H is a subgroup of G then prove that f(H) is a subgroup of G' (ii) If K is a subgroup of G then prove that $f^{-1}(K)$ is a subgroup of G.

(P.T.O)

17. Prove that every finite integral domain is a field.

18. Prove that every euclidean ring is a principal ideal domain.

PART – C

Answer any TWO questions **(2 x 20 = 40 marks)**

19. (i) If H and K are finite subgroups of a group G then prove that O (HK) = $\frac{O(H)O(K)}{O(H\cap K)}$.

(ii) If f is a homomorphism of a group G into a group G' then prove that kernel of f is a normal subgroup of G. (12+8)

20. (i) State and prove Lagrange's theorrem.

(ii) Suppose that N and M are two normal subgroups of G and that $N \cap M = (e)$. Show that for any n $\in$ N, m $\in$ M, mn = nm. (12+8)

21. (i) State and prove fundamental theorem of homomorphism for a group.

(ii) Prove that the intersection of two subrings of a ring R is a subring of R. (14+6)

22. (i) State and prove unique factorization theorem.

(ii) Prove that the chracteristic of an integral domain D is either zeo or a prime number. (12+8)

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